Application of the Lorentz-Transform Technique to Meson Photoproduction

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We show that the Lorentz integral transform (LIT) technique which has been successfully applied to photoreactions in light nuclei can also be applied to photoreactions involving particle production. A simple model where results are easily calculable in the traditional fashion is used to test the technique. Specifically we compute inclusive π^+ photoproduction from deuterium for photon energies less than 200 MeV using a Yamaguchi model for the NN interaction. It is demonstrated that although the response functions for inclusive meson production do not have favourable asymptotic behavior one can nontheless extract them by inversion of the transform. The implication is that one can treat realistic problems of photo-meson production including all final state interactions by means of the LIT technique.

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I. INTRODUCTION

In a series of papers [1, 2, 3, 4, 5, 6, 7] it has been demonstrated that the LIT technique allows a convenient calculation of inclusive photoreaction cross sections wherein final state interactions are fully included. Further, the technique has been extended [8, 9] to exclusive reactions. A merit of the technique is that the calculation of continuum wave functions is avoided. In fact the differential equations to be solved are inhomogeneous and have solutions bounded at infinity. The work cited above is based on non-relativistic quantum mechanics and a nucleons-only subspace.

In order to treat meson photoproduction from nuclei it is desirable to see if the LIT technique can be extended to a larger subspace i.e. one including nucleons and mesons. This would enable the full inclusion of both meson-nucleon and NN interactions in the production calculations. As a test we consider inclusive photoproduction of low energy (<40 MeV) π^+ mesons from deuterium. A traditional calculation of this process was reported by Dressler, MacDonald, and O'Connell[10] and by Gupta, Anand, and Bhasin [11]. Both of these groups employed a simple Yamaguchi potential [12] with parameters chosen to account for deuteron binding and low energy NN scattering properties. We adopt this model with the parameters used in [10]. Thus this example attempts to apply the LIT technique in the NNπ subspace where for simplicity only an NN interaction is included. Although no $\pi-$N scattering is contained in this model, this is not essential for testing the method. Our results will show that it is indeed possible to apply the LIT method to this problem and to extract the total photoproduction cross section. We should point out that our restriction to a simple NN potential for this test calculation does not modify these conclusions. This is clearly seen from the series of papers [1, 2, 3, 4, 5, 6, 7] where a range of NN potentials, from simple phenomenological models to modern realistic potentials, have been used in 3-nucleon and 4-nucleon problems. In no case did the complexity of the NN potential model have any effect on the accuracy or implementation of the LIT technique. With assurance that the formalism works the addition of a meson-nucleon interaction can be viewed as a technical point, the consequence of which would be more complicated numerics.

Low energy photoproduction is described by the Kroll-Ruderman [13] operator

$$\mathcal{H}_{\rm int}(x) = -i\, e \left(\frac{f}{m_\pi}\right) \sum_j \hat{\epsilon}_\lambda \cdot \vec{\sigma}(j)\, \tau_-(j)\, e^{-i\,k\cdot x}\, \phi_+(x)\, \delta(\vec{x}-\vec{x}_j) \quad . \tag{1}$$

where $\vec{k}$ and $\hat{\epsilon}_\lambda$ denote the incident photon momentum and polarization vectors , respectively, and $\phi_+(x)$ denotes the meson field operator

$$\phi_+(x) = \frac{1}{\sqrt{8\pi^3}} \int \mathrm{d}^3q\, \frac{1}{\sqrt{2q_0}} \left[e^{iq\cdot x}\, a_+^\dagger(\vec{q}) + e^{-iq\cdot x}\, a_-(\vec{q})\right] \tag{2}$$

with q_0 the energy of the meson and $a^\dagger$ and a are the usual creation and annihilation operators. Here e is the positive elementary charge, m_π is the π^+-meson mass, f is the π-N coupling constant, $\vec{\sigma}$ denotes the Pauli spin matrix vector and τ_- the isospin operator. Dressler *et al.* [10] show that the KR term gives nearly the entire cross section for pion energies in the range considered here. Due to cancellations other terms in the production operator i.e. terms required by gauge invariance, do not contribute significantly at low pion energies. With $E_{\rm cm}$ as the incident c.m. energy the response function $\mathcal{R}(E_{\rm cm})$ for this process is

$$\mathcal{R}(E_{\rm cm}) = \frac{1}{6} \sum_{M_d,\lambda} \sum_f \left| \langle\, f \,|\, \tilde{\mathcal{O}}(\vec{k},\lambda) \,|\, D, M_d \,\rangle \right|^2 \delta(E_{\rm cm} - E_f) \quad , \tag{3}$$

where $|\, D, M_d \,\rangle$ denotes the deuteron ground state with polarization M_d and

$$\tilde{\mathcal{O}}(\vec{k},\lambda) = \int \mathrm{d}^3 q \; \mathcal{O}(\vec{k},\lambda,\vec{q}) \, a_+^\dagger(\vec{q}) \tag{4}$$

with

$$\mathcal{O}(\vec{k},\lambda,\vec{q}) = -i\, e \left(\frac{f}{m_\pi} \right) \frac{1}{\sqrt{8\pi^3}} \frac{1}{\sqrt{2q_0}} \sum_j \hat{\epsilon}_\lambda \cdot \vec{\sigma}(j)\, \tau_-(j)\, e^{i\, \vec{x}_j \cdot (\vec{k}-\vec{q})} \quad . \tag{5}$$

In Eq. (3) above $|\, f \,\rangle$ denotes the wave function of the relative motion of the $nn\pi^+$ system with energy E_f, and $\sum_f$ indicates an integration over all relative momenta and a sum over all final nucleon spins. The inclusive cross section is related to $\mathcal{R}(E_{\rm cm})$ by

$$\sigma(E_{\rm cm}) = \frac{2\pi^2}{k} \mathcal{R}(E_{\rm cm}) \quad . \tag{6}$$

Relative momenta in the final state are taken as

$$\vec{p}_x = (\vec{p}_1 - \vec{p}_2)/2 \tag{7}$$

$$\vec{p}_y = -\frac{m_\pi}{M} (\vec{p}_1 + \vec{p}_2) + \frac{2m_n}{M} \vec{p}_\pi \tag{8}$$

where $\vec{p}_1, \vec{p}_2$, and $\vec{p}_\pi$ are the momenta of the final state neutrons and pion, respectively, and M is the total mass $2m_n + m_\pi$ with the mass of the neutron m_n. As implied by this separation we are restricting ourselves to non-relativistic kinematics for both the nucleons and the pion. This allows the separation of the hamiltonian into Jacobi coordinates for the NNπ three body system. In terms of these quantities the energy conserving δ-function appearing in $\mathcal{R}(E_{\rm cm})$ takes the detailed form

$$\delta(E_{\rm cm} - E_f) = \delta \left(E_{\rm cm} - (2m_n + m_\pi) - \frac{p_x^2}{m_n} - \frac{p_y^2}{2\mu} \right) \quad , \tag{9}$$

where μ is the reduced two-neutron-pion mass. Finally it is more convenient to use the c.m. energy above threshold

$$\mathcal{W} = k + \frac{k^2}{2m_d} - (2m_n + m_\pi - m_d) \geq 0$$

where m_d is the mass of the deuteron. The LIT, referred to hereafter as the transform, of the response function $\mathcal{R}(\mathcal{W})$ is then defined with σ_R, $\sigma_I > 0$ as

$$L(\sigma_R,\sigma_I) = \int_0^\infty \mathrm{d}\mathcal{W} \, \frac{\mathcal{R}(\mathcal{W})}{(\mathcal{W}-\sigma_R)^2 + \sigma_I^2} \tag{10}$$

$$= \frac{1}{6} \sum_{M_d,\lambda} \int_0^\infty \mathrm{d}\mathcal{W} \, \langle\, D, M_d \,|\, \tilde{\mathcal{O}}^\dagger(\vec{k},\lambda) \, \frac{\delta(\mathcal{W} - H)}{(H-\sigma_R)^2 + \sigma_I^2} \, \tilde{\mathcal{O}}(\vec{k},\lambda) \,|\, D, M_d \,\rangle. \tag{11}$$

Because of the relation of k to $\mathcal{W}$, a straightforward integration of the above equation would leave the operator $\tilde{\mathcal{O}}(\vec{k},\lambda)$ depending in a complicated way on the Hamiltonian H. Therefore, we proceed by setting $\vec{k}$ appearing in the operator

$\tilde{\mathcal{O}}$ to a constant arbitrarily chosen "pseudo-momentum" $\vec{k}_{\rm p}$. As a result we introduce a new transform $L_{k_{\rm p}}$, which depends on $k_{\rm p}$ and takes the form

$$L_{k_{\rm p}}(\sigma_R,\sigma_I) \;=\; \frac{1}{6}\sum_{M_d,\lambda} \langle\, D, M_d \,|\, \tilde{\mathcal{O}}^\dagger(\vec{k}_{\rm p},\lambda)\, \frac{1}{(H-\sigma_R)^2+\sigma_I^2}\, \tilde{\mathcal{O}}(\vec{k}_{\rm p},\lambda)\,|\, D, M_d\,\rangle \tag{12}$$

$$\;=\; \frac{1}{6}\sum_{M_d,\lambda} \langle\, \tilde{\psi}_{M_d,\lambda}(\sigma_R,\sigma_I,k_{\rm p}) \,|\, \tilde{\psi}_{M_d,\lambda}(\sigma_R,\sigma_I,k_{\rm p})\,\rangle \quad , \tag{13}$$

where the Lorentz function $\tilde{\psi}$ is solution of the inhomogeneous equation

$$(H-\sigma_R+i\sigma_I)\,|\,\tilde{\psi}_{M_d,\lambda}(\sigma_R,\sigma_I,k_{\rm p})\,\rangle = \tilde{\mathcal{O}}(\vec{k}_{\rm p},\lambda)\,|\, D, M_d\,\rangle \tag{14}$$

with

$$\langle\, \vec{p}_x, \vec{p}_y \,|\, H \,|\, \vec{p}_x{}', \vec{p}_y{}'\,\rangle = \left[\left(\frac{p_x^2}{m_n}+\frac{p_y^2}{2\mu}\right)\delta(\vec{p}_x-\vec{p}_x{}') + V(\vec{p}_x,\vec{p}_x{}')\right]\delta(\vec{p}_y-\vec{p}_y{}') \quad . \tag{15}$$

The inverse to the transform $L_{k_{\rm p}}(\sigma_R,\sigma_I)$ will lead to a corresponding response function $\mathcal{R}_{k_{\rm p}}(\mathcal{W})$ and in turn will yield the correct cross section for $k_{\rm p}=k$. Consequently for each photon energy the calculation of the transform and its inversion must be repeated.

As mentioned earlier the simple model here only includes an NN potential, i.e., $V(\vec{p}_x,\vec{p}_x{}')$. The kinetic energy of the meson with respect to the nn pair appears in H as the term proportional to p_y^2. Meson rescattering could be included by adding to H appropriate potentials. Although that would considerably complicate the numerical aspect, it would not affect the question posed here, namely: can the LIT be inverted when the response function arises from a particle production process? A problem is the slow fall-off of the response function for high $\mathcal{W}$ values. From the definition of the transform it is clear that the response must behave asymptotically like $\mathcal{W}^{1-x}$ where $x>0$ in order for the integral to converge. In the case of nuclear photoabsorbtion without pion production the response function falls off very rapidly for increasing $\mathcal{W}$ resulting in $L(\sigma_R,\sigma_I)$ also falling rapidly for large σ_R. Inversion then gives an accurate account of $\mathcal{R}(\mathcal{W})$ over its entire range by calculating $L(\sigma_R,\sigma_I)$ for a finite number of σ_R values. That the response function for inclusive photoproduction may have a significantly different asymptotic behavior can be seen from the basic Kroll-Rudermann [13] cross-section for $p(\gamma,\pi^+)n$

$$\sigma = 2\,\alpha\,\left(\frac{f}{m_\pi}\right)\left(\frac{q}{k}\right)\frac{E_n(q)\,E_p(k)}{E_{\rm cm}^2} \tag{16}$$

where E_N are the nucleon energies, $E_{\rm cm}$ is the c.m. energy $k+E_p(k)$ and α the fine structure constant. This cross section approaches a constant for large $E_{\rm cm}$ and the response $\mathcal{R}(E_{\rm cm})$ rises linearly in $E_{\rm cm}$. In the next section it will be seen that the pion production response function for a finite nucleus is tempered at large $E_{\rm cm}$ by structure effects. Nevertheless it still rises over a large energy region thereby requiring a different approach for the inversion of the transform.

II. RESULTS

Details of the model and the parameters used are given in the Appendix. There it is seen that the vector $|\,\tilde{\psi}_{M_d,\lambda}(\sigma_R,\sigma_I,k_{\rm p})\,\rangle$ can also be labeled by the final neutron-neutron spin and that the response function is therefore a sum of singlet and triplet contributions. The separable potential used here only has scattering in the spin $S=0$, isospin $T=1$ channel while the $S=1$, $T=1$ final state consisting of odd partial waves is non-interacting. Fig. 1 shows the transform $L_{k_{\rm p}}(\sigma_R,\sigma_I)$ for the case $\sigma_I=k_{\rm p}=10$ MeV. One notes that $L_{k_{\rm p}}$ reaches a maximum at $\sigma_R\approx 200$ MeV and then falls very slowly with increasing σ_R. Also shown in this figure is the response function $\mathcal{R}_{k_{\rm p}}(\mathcal{W})$ for $k_{\rm p}=10\,$MeV. The response function behaves similarly to L as one would expect since $L(\sigma_R,\sigma_I)$ samples $\mathcal{R}$ mainly from a region centered at $\mathcal{W}=\sigma_R$. We note that $\mathcal{R}_{k_{\rm p}}(\mathcal{W})$ does not rise linearly with energy as in the case of $p(\gamma,\pi^+)n$ but falls off slowly with energy after reaching a maximum. Unfortunately, the fall-off of L with σ_R is so slow and covers such a large energy range that inverting it to obtain $\mathcal{R}(\mathcal{W})$ over the full range, as was possible in the earlier photoabsorption calculations, is not only very difficult but largely physically meaningless because our model is only valid for non-relativistic mesons. In fact the present model is only sensible for energies $\mathcal{W}$ not exceeding approximately 40 MeV which corresponds to the maximal pion energy still being non-relativistic. Our aim then is to calculate L

only for the segment $0 \leq \sigma_R \leq 40$ MeV and then invert it to extract $\mathcal{R}_{k_\mathrm{p}}(\mathcal{W})$ for $\mathcal{W}$ in the same energy range. Since $\mathcal{R}$ can be calculated directly the error can be easily assessed.

The inversion process we use has already been described in [3] but a brief account is as follows. The response function $\mathcal{R}_{k_\mathrm{p}}(\mathcal{W})$ is written as a sum

$$\mathcal{R}_{k_\mathrm{p}}(\mathcal{W}) = \sum_{i=1}^{N} \alpha_i \, \mathcal{W}^{S+2} \, e^{-\beta_i \mathcal{W}} \equiv \sum_{i=1}^{N} \mathcal{R}^i_{k_\mathrm{p}}(\mathcal{W}) \tag{17}$$

and the parameters α_i and β_i are determined by fitting

$$L^{\mathrm{fit}}_{k_\mathrm{p}}(\sigma_R, \sigma_I) = \sum_{i=1}^{N} \int_0^\infty \mathrm{d}\mathcal{W} \, \frac{\mathcal{R}^i_{k_\mathrm{p}}(\mathcal{W})}{(\mathcal{W} - \sigma_R)^2 + \sigma_I^2} \tag{18}$$

to L as computed from Eqs. (12)-(13) of the Appendix. Note that the threshold behavior $\mathcal{W}^{S+2}$, where S is the total spin, is appropriate to the photoproduction process [11] and differs slightly from the form used in the earlier photoabsorption calculations. It turns out however that the quality of fit is nearly independent of which threshold behaviour is used. Fig. 2 shows the quality of the fit obtained using $N = 10$. With the parameters so determined one can calculate the response functions and finally the inclusive cross section. As mentioned earlier this model problem is simple enough that the response functions are easily calculated in the traditional manner [10, 11]. Fig. 3 shows the relative error between the response function as calculated from the LIT technique ($\mathcal{R}_{\mathrm{fit}}$) and the one calculated in the traditional manner. Over most of the energy range between $0 \leq \mathcal{W} \leq 40$ MeV the relative error is in the 1% range. However near the threshold, i.e. $\mathcal{W} < 2$ MeV the response function tends to zero for $\mathcal{W} \longrightarrow 0$ and limited numerical accuracy produces an exaggerated relative error in this region. These effects are too small to be visible in a plot of the cross section however. Finally our computed cross section along with data of Booth *et al.* [14] is shown in Fig. 4. The results of using the LIT technique are indistinguishable from the traditional methods in [10, 11].

It is instructive to see the differences in the response functions that occur if one uses transforms calculated in different ranges of σ_R. One expects that because of the nature of the Lorentz transform that one should only have to calculate the transform up to $\sigma_R = 40\,\mathrm{MeV}$, as was done above, if the response function was only required in the energy range $0 \leq \mathcal{W} \leq 40$ MeV. To show this we calculate the transform in four ranges $0 \leq \sigma_R \leq \sigma_{R_{\max}}$ where $\sigma_{R_{\max}} = 20, 40, 60$, and $70\,\mathrm{MeV}$. The respective response functions obtained by inverting each of these transforms is denoted by $\mathcal{R}_{k_\mathrm{p}}(\mathcal{W}|\sigma_{R_{\max}})$. Fig. 5 shows the error in the $\sigma_{R_{\max}}=$ 20,40,60 MeV cases relative to the $\sigma_{R_{\max}}=$ 70 MeV case. One notes that the $\sigma_{R_{\max}}=$ 20 MeV case only is accurate for energies up to 20 MeV, that the $\sigma_{R_{\max}}=$ 40 MeV case has a less than 1% error at $\mathcal{W}=$ 40 MeV, and that the $\sigma_{R_{\max}}=$ 60 MeV case has only a very small error up to 60 MeV.

III. CONCLUSIONS

We have shown that the LIT technique can be used to calculate inclusive meson photoproduction cross sections in the non-relativistic regime. Rather than fitting the transform over its entire range as was possible in earlier photoabsorbtion calculations one fits here only the low energy segment to obtain the low energy response functions. The model calculation used here shows that the response functions thus obtained are as accurate as numerical techniques will allow. Our next step will be to add a pion-nucleon interaction to the Hamiltonian in order to take account of pion scattering effects. The dynamical model of Darwish, Arenhoevel, and Schwamb [15] would provide a benchmark calculation against which to further check our method. It would also be of interest to apply these types of calculations to higher A nuclei such as ^{3}He, ^{3}H, or ^{4}He. The LIT technique extends readily to these cases as well as being able to handle realistic potentials including Coulomb effects. One should expect to be able to study meson-rescattering effects with realistic nuclear models in a variety of light nuclei.

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Appendix

For the NN interaction we use the separable model of Y. Yamaguchi [12], which is a pure S-wave interaction

$$\begin{aligned} V(\vec{p},\vec{p}\,') &= -\lambda_0\, g_0(\vec{p})\, g_0(\vec{p}\,')\, \frac{1}{4}\, (1-\vec{\sigma}_1\cdot\vec{\sigma}_2) \\ &\quad -\lambda_1\, g_1(\vec{p})\, g_1(\vec{p}\,')\, \frac{1}{4}\, (3+\vec{\sigma}_1\cdot\vec{\sigma}_2) \quad , \end{aligned} \tag{1}$$

$$g_S(\vec{p}) = \frac{1}{\vec{p}^{\,2}+\beta_S^2} \quad S\in\{0,1\}\,. \tag{2}$$

Here the labels 0 and 1 refer to the spin-singlet and spin-triplet parts respectively. The following more up to date constants for this model have been taken from [10]:

$$\alpha = 0.2316\,\mathrm{fm}^{-1} \quad , \quad \beta_0 = 1.129\,\mathrm{fm}^{-1} \quad , \quad \beta_1 = 1.392\,\mathrm{fm}^{-1} \quad , \tag{3}$$

$$\lambda_0 = 0.02774\,\mathrm{fm}^{-2} \quad , \quad \lambda_1 = \frac{\beta_1\,(\alpha+\beta_1)^2}{m_n\,\pi^2} \quad . \tag{4}$$

These constants fit the deuteron binding energy, the experimental values for the singlet and triplet scattering lengths and the singlet effective range.

Using this separable NN interaction one obtains for the deuteron ground state

$$\psi_d(\vec{p}) = \frac{\sqrt{\alpha\,\beta_1\,(\alpha+\beta_1)^3}}{\pi}\, \frac{1}{(\vec{p}^{\,2}+\alpha^2)\,(\vec{p}^{\,2}+\beta_1^2)} \tag{5}$$

and a binding energy of 2.224 MeV.

The Lorentz function $\tilde{\psi}_{M_d,\,\lambda}(\sigma,\,k_{\mathrm{p}},\,\vec{p}_x,\,\vec{p}_y)$ can be decomposed into its spin components as

$$\tilde{\psi}_{M_d,\,\lambda}(\sigma,\,k_{\mathrm{p}},\,\vec{p}_x,\,\vec{p}_y) = \sum_{S=0,1} \tilde{\psi}^S_{M_S}(\sigma,\,k_{\mathrm{p}},\,\vec{p}_x,\,\vec{p}_y) \tag{6}$$

where $M_S = M_d + \lambda$ with M_d and λ the deuteron and photon polarization, respectively, and the function $\tilde{\psi}^S_{M_S}(\sigma,\,k_{\mathrm{p}},\,\vec{p}_x,\,\vec{p}_y)$ is a solution of

$$\begin{aligned} \left[\frac{p_x^2}{m_n}+\frac{p_y^2}{2\mu}-\sigma\right] \tilde{\psi}^S_{M_S}(\sigma,\,k_{\mathrm{p}},\,\vec{p}_x,\,\vec{p}_y) - \lambda_S\, g_S(p_x)\, C^S_{M_S}(\vec{p}_y,\,\vec{\Delta}) \\ = \Xi^S_{M_S,M_d,\lambda}\, \mathcal{F}^S_{\vec{\Delta}}(\vec{p}_x) \quad , \quad S\in\{0,1\} \quad , \end{aligned} \tag{7}$$

where

$$C^S_{M_S}(\vec{p}_y,\,\vec{\Delta}) = \int \mathrm{d}^3p\, g_S(p)\, \tilde{\psi}^S_{M_S}(\sigma,\,k_{\mathrm{p}},\,\vec{p},\,\vec{p}_y) \quad , \tag{8}$$

$$\mathcal{F}^S_{\vec{\Delta}}(\vec{p}) = \psi_d(\vec{p}-\vec{\Delta}) + (-1)^S \psi_d(\vec{p}+\vec{\Delta}) \quad , \tag{9}$$

$$\Xi^S_{M_S,M_d,\lambda} = -i\,\frac{3\sqrt{2}\,e\,f}{m_\pi}\,(-1)^{1+S-M_S}\,\sqrt{2S+1} \begin{pmatrix} S & 1 & 1 \\ -M_S & \lambda & M_d \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & S & \frac{1}{2} \\ 1 & \frac{1}{2} & 1 \end{Bmatrix} \quad . \tag{10}$$

We take $f^2/4\pi$=0.078.

The constant $C^S_{M_S}(\vec{p}_y,\,\vec{\Delta})$ with respect to p_x is

$$C^S_{M_S}(\vec{p}_y,\,\vec{\Delta}) = m_n\,\Xi^S_{M_S,M_d,\lambda} \int \mathrm{d}^3p\, \frac{g_S(p)\,\mathcal{F}^S_{\vec{\Delta}}(\vec{p})}{p^2+\gamma^2-m_n\,\sigma} \left[1-\frac{m_n\,\pi^2\,\lambda_S}{\beta_S\,\left(\beta_S+\sqrt{\gamma^2-m_n\,\sigma}\right)^2}\right]^{-1} \tag{11}$$

with $(m_n/\mu)/2\,\vec{p}_y^{\,2} \equiv \gamma^2(\vec{p}_y)$. In the case of $S=1$ this constant vanishes.

The solution of the Lorentz-equation therefore is

$$\tilde{\psi}^S_{M_S}(\sigma,\, k_\mathrm{p},\, \vec{p}_x,\, \vec{p}_y) = \frac{m_n\, \Xi^S_{M_S,M_d,\lambda}\, \mathcal{F}^S_{\vec{\Delta}}(\vec{p}_x) + m_n\, \lambda_S\, g_S(p_x)\, C^S_{M_S}(\vec{p}_y,\, \vec{\Delta})}{p_x^2 + \gamma^2 - m_n\, \sigma} \tag{12}$$

and the transform $L_{k_\mathrm{p}}(\sigma_R, \sigma_I)$ is

$$L_{k_\mathrm{p}}(\sigma_R, \sigma_I) = \frac{1}{6} \sum_{S,\, M_d,\, \lambda} \langle \tilde{\psi}^S_{M_S} \,|\, \tilde{\psi}^S_{M_S} \rangle \quad . \tag{13}$$

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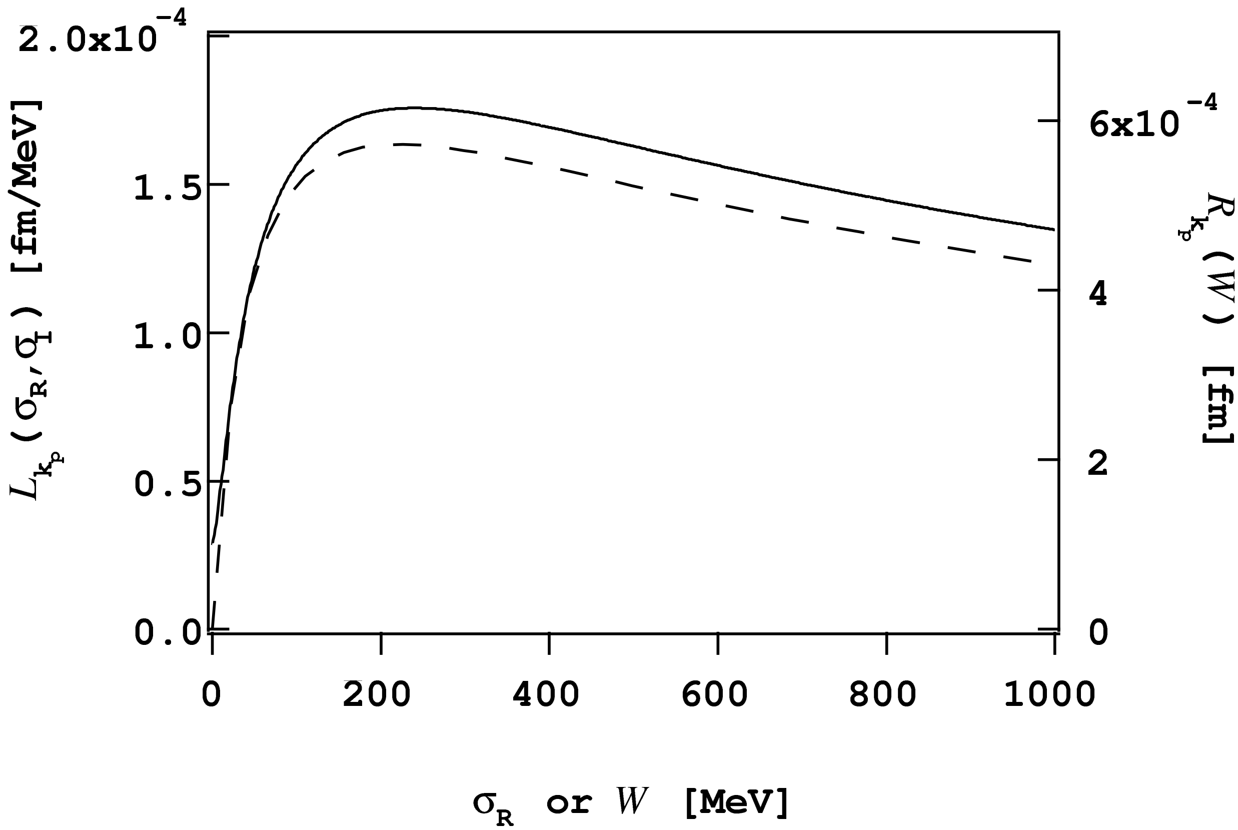

FIG. 1: $L_{k_{\rm p}}(\sigma_R, \sigma_I)$ (solid line) and $\mathcal{R}_{k_{\rm p}}(\mathcal{W})$ (dashed line) shown for $\sigma_I = k_{\rm p} = 10$ MeV.

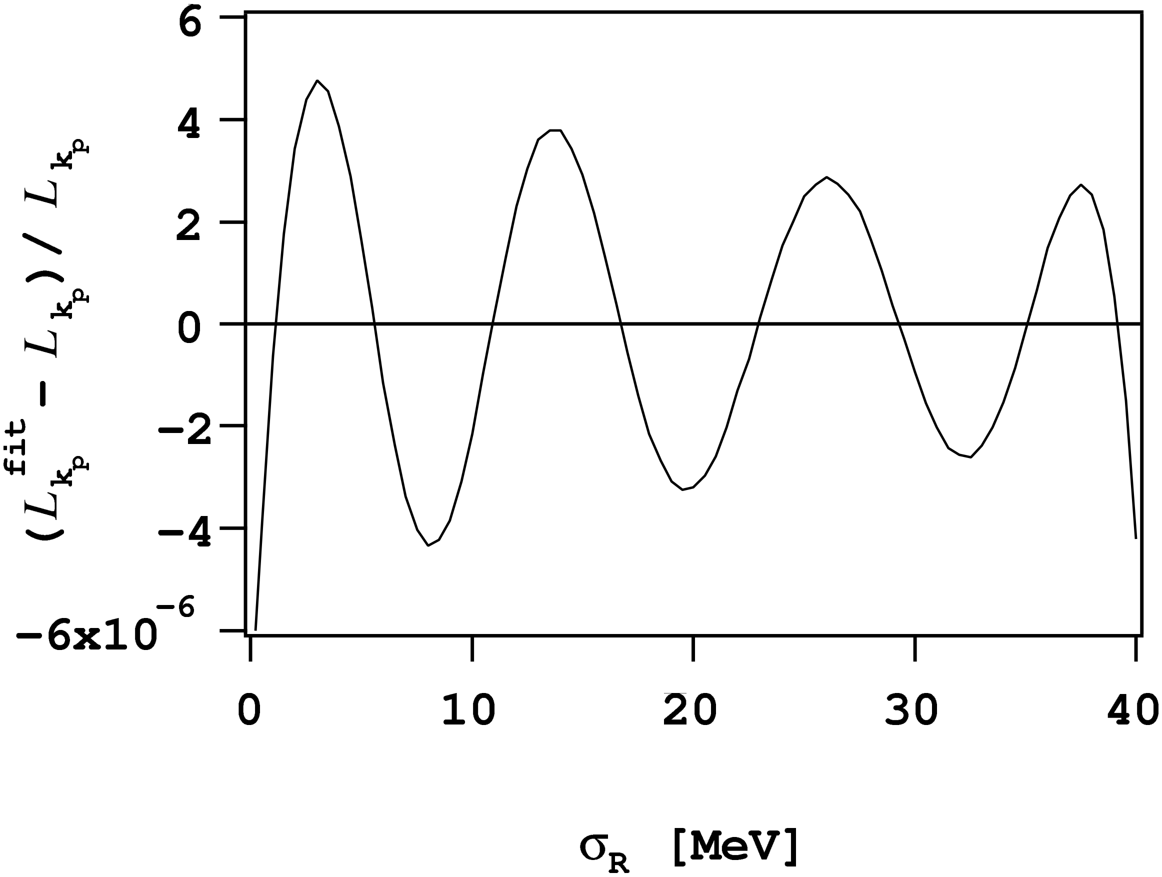

FIG. 2: Relative error of the fit $L^{\rm fit}_{k_{\rm p}}(\sigma_R, \sigma_I)$ compared to the computed transform $L_{k_{\rm p}}(\sigma_R, \sigma_I)$. Here we have used $\sigma_I = k_{\rm p} = 10$ MeV for illustration.

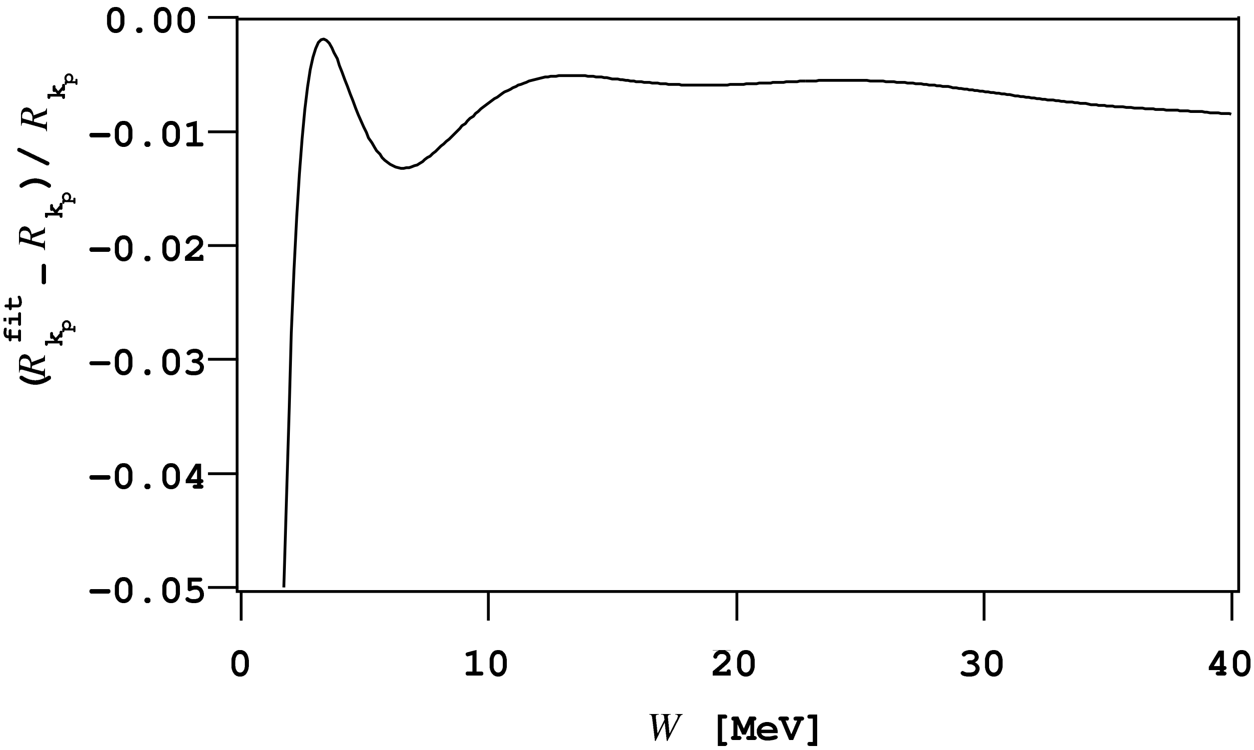

FIG. 3: Relative error between the response function $\mathcal{R}^{\rm fit}_{k_{\rm p}}(\mathcal{W})$ and $\mathcal{R}_{k_{\rm p}}(\mathcal{W})$ computed in the traditional method for $\sigma_I = k_{\rm p} = 10$ MeV.

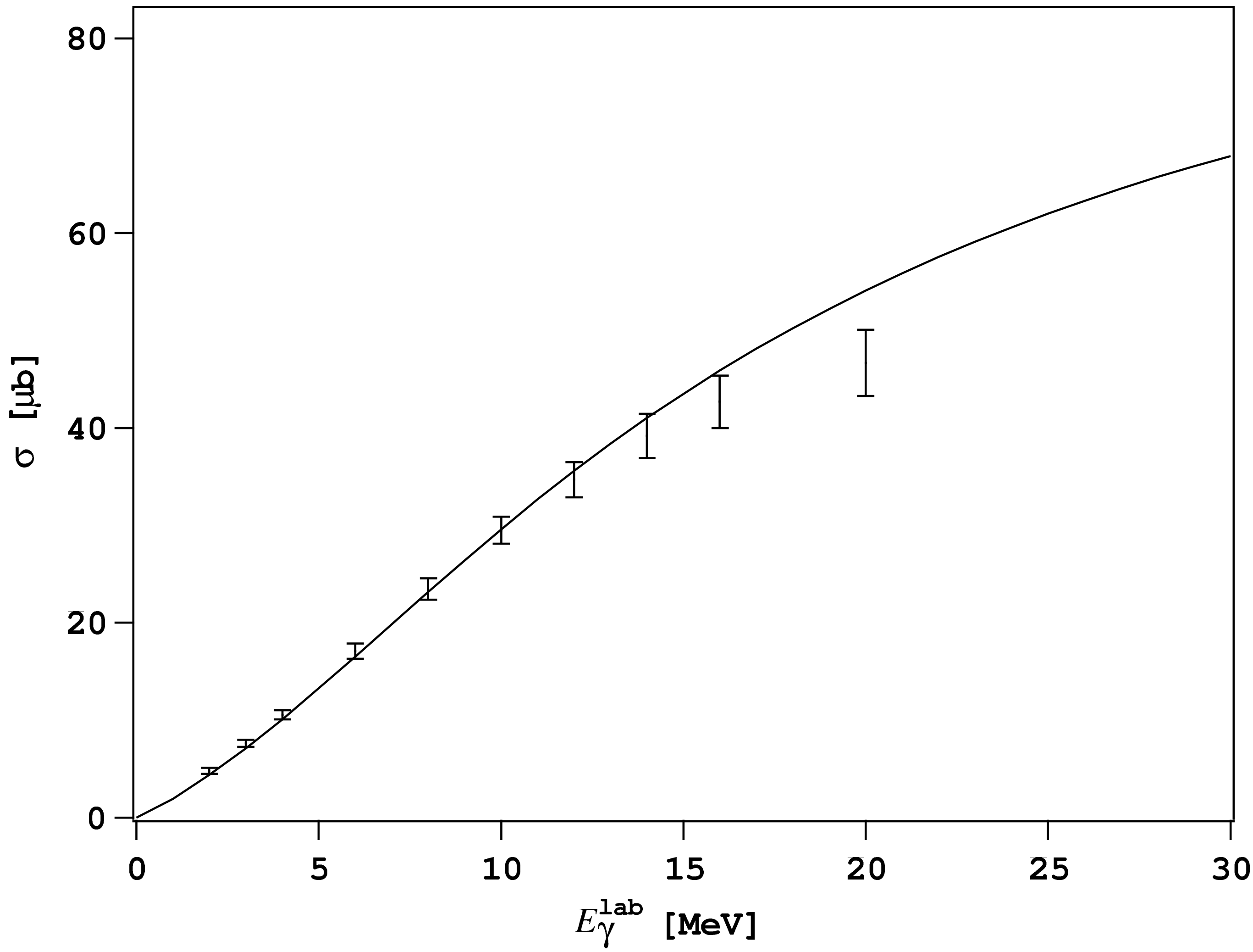

FIG. 4: Calculated total cross section for D$(\gamma,\pi^+)nn$ shown together with the data of [14]. The cross sections calculated either by the LIT technique or the traditional method [10, 11] are indistinguishable.

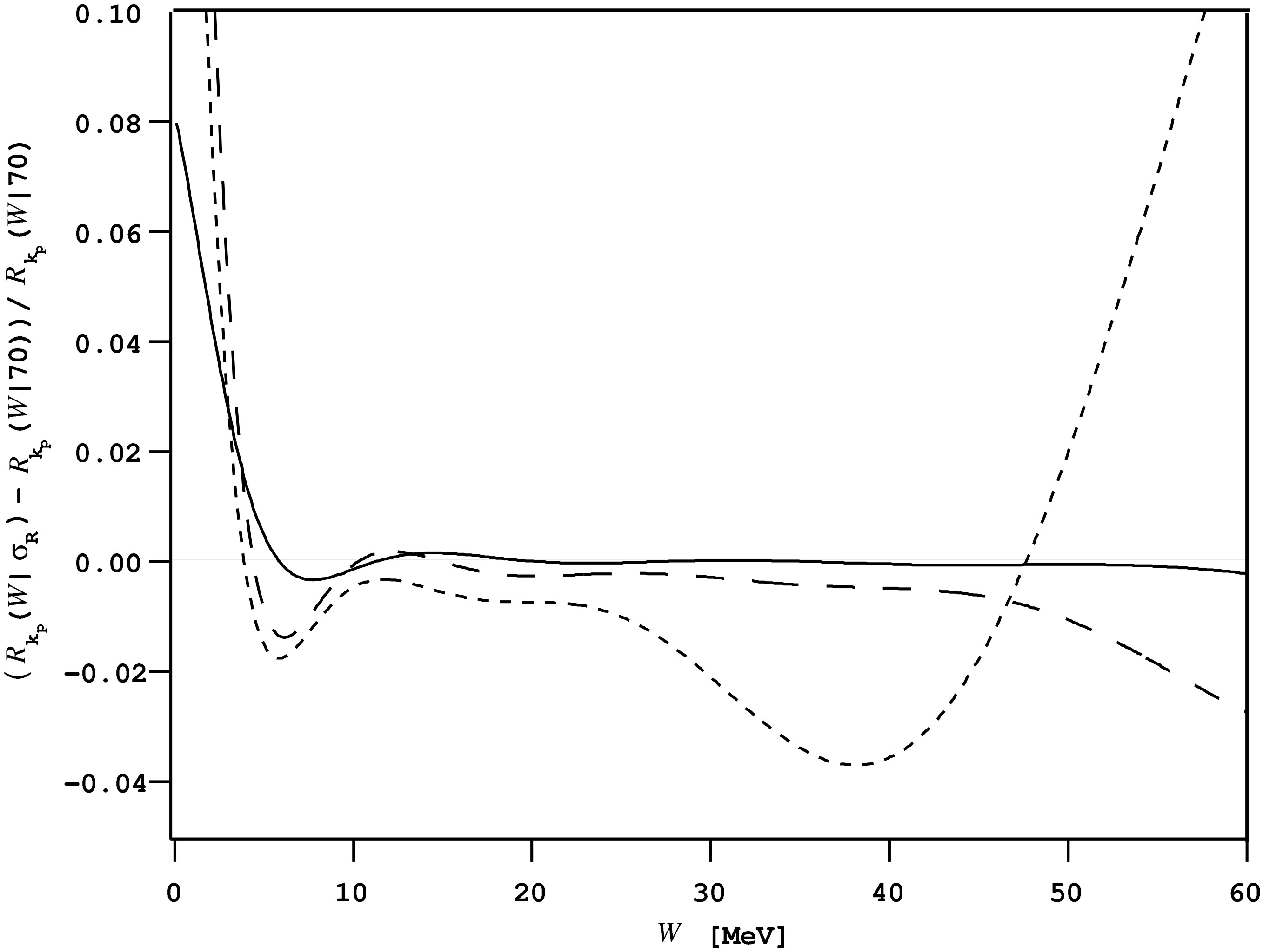

FIG. 5: Relative errors in response functions for $\sigma_I = k_{\mathrm{p}} = 10$ MeV with respect to the $\sigma_{R_{\max}}$=70 MeV case: $\sigma_{R_{\max}}$= 60 MeV (solid line), $\sigma_{R_{\max}}$= 40 MeV (dashed line), $\sigma_{R_{\max}}$= 20 MeV (dotted line).